Optimization techniques for animal diet formulation

By

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Ration formulation is one of the basic needs of animal yield industries. Performance and development of animal is directly dependent on diet intake of animal. To meet the animal's requirement at a particular stage of production, it is very important to formulate the diet efficiently. Form of food, which is taken by animal on daily basis, is termed as ration and formulation of ration is concerned to combine different nutrient ingredients in such an efficient manner that it can provide sufficient amount of energy and nutrition to animal at different stages of metabolism. Different classes of animal have different requirements of energy and ration should be formulated in such a way that energy requirement of animal must be fulfilled. Formulation of ration is a difficult task as it should select a combination of feed ingredients that adequately meet stated nutrients and other requirements of animal. Main objective of ration formulation is to achieve a specific satisfactory nutrient level of animal species at least cost.

For the last thirty years, linear programming has been used as a tool for ration formulation and it refers to as least cost formulation technique. Firstly, Waugh(1951) has used linear programming as a tool for optimizing livestock ration. Linear programming is used to balance the nutrient ingredients included in the diet and minimize the cost of ration. A number of models have been derived for different objective of study and constrains. The LP model can be solved for a complicated set of nutrient requirements to give a relatively well-balanced ration [VandeHaar and Black, M. J., 1991]. The principal objective in the application of LP to feed formulation is the production of least cost rations that will produce satisfactory results. A nutrition program was developed for high producing dairy herds to attain efficient and profitable levels of milk production [Sklan, D. and Dariel, I., 1993]. In 2002, Darmon has used linear programming as a tool to optimize human nutrition. A model was developed to represent the efficiency of nutrient use and its relationship to profitability on dairy farms [Tedeschi, L. O, 2004]. A cost analysis spreadsheet and validation of that spreadsheet on milking and custom heifer operations was developed [Guevara V.R., 2004]. Lead factors are used in computerized ration formulation programs developed at Virginia Tech to increase milk production above a herd or group average for which total mixed rations are formulated for group feeding [Stallings, C. C; Mcgilliard, M. L]. Chance constrained programming is used to formulate commercial feeds for animals [Britt, J. S; Thomas, R. C; Speer, N. C; Hall, M. B., 2003]. A stochastic-linear program Excel workbook was developed that consisted of 2 worksheets illustrating
linear and stochastic program approaches. Both approaches used the Excel Solver add-in. Excel spreadsheet was set up so that the calculated margin of safety (MOS) value, according to the requested probability, was the same for both the linear and stochastic programs. A multiple-objective programming (MOP) model was applied to the feed formulation process with the objectives of minimizing nutrient variance and minimizing ration cost. A study was conducted to introduce a dual model in an original linear program to obtain the shadow prices of resources that take part in optimization, in feed formulation. The shadow prices of nutrients resource showed degrees of influence of a diet's least cost when increasing or decreasing expected diet nutrient b values of a diet. The higher the shadow prices of a nutrient resource, the more obvious its influence on least cost. When the shadow price of a kind of resource equals to zero, it means that reaching this nutrient value does not have an influence on a special diet least cost within a particular value. [Xiong BenHai, Luo QingYao, Pang ZhiHong].

While constructing linear programming model, some assumptions are made for simplification of model. Following assumptions are made for linear programming model: Certainty, Additivity, Linearity and Divisibility. While formulating linear model, it is assumed that all parameters of the model and consumption of resources are known and may be constant under the assumption of certainty. Additivity assumption says that the value of the objective function and the total amount of each resource is proportional to the value of each decision variable. The solution values of decision variables are allowed to assume continuous and non-negative values. These assumptions indicate that the objective function and all the constraints must be characterized by linear relationship among decision variables in linear programming model. It is also assumed that the linear programming models are completely deterministic. General linear programming model with n decision variables and m constraints can be stated as:

\[
\text{Optimize} \quad Z = \sum_{j=1}^{n} c_j x_j
\]

Subject to the linear constraints:
\[
\sum_{j=1}^{n} a_{ij} x_j \leq = \geq b_i , \quad i = 1,2, \ldots, m
\]

And
\[
x_j \geq 0 , \quad j = 1,2, \ldots, n
\]

Solution of linear programming consists of a number of linear equations, which has constraint variable in Left hand side and values on right hand side. Right hand variable is called row variable, while variable on left hand side is called column variable. While formulating linear programming problem, only one variable may appear on the RHS. While row variables, if there are more than one, may appear on the LHS along with column variables, column variables may not appear on the RHS of any problem. The same variable may not appear on the RHS of more than one equation and the coefficient of the variable on the RHS of an equation must be 1.0.

While formulating ration by linear programming, first of all available raw material and feed ingredients are selected which are to be included in ration and then a set of constraints on feed ingredient is set up. Objective function is then formulated according to the available feed ingredients and constraint to achieve the objective of least cost ration. A number of computer program and software are available for this purpose. FeedLIVE, This software is meant for least cost feed formulation and it is bilingual for Thai
and English language. Feed MU is a simple feed formulation software, which is based on trial-and-error method and simplex method of linear programming. FeedMU is upgraded and new version is FeedMU 2, which uses .NET framework 2.0 technology. FEED FORMULATION is software which is used to formulate the optimum diet. It uses two functions optimize and analysis. Optimize tool uses Linear Programming for optimizing Feed Formulation at Least Cost. WinFeed is very simple and straightforward software, which prepares least cost formula diets for ruminants and non-ruminants by using linear and stochastic programming method. ECOMIX, BESTMIX, OPTIMIX are other available software for feed formulation.

All these software and solutions are based on linearity assumption which in practice may not be always true. Formulation of least cost ration by linear programming involves omitting ingredients with an inclusion rate below some fixed threshold and rounding other ingredients to realistic weighing quantities. It is possible to incorporate these constraints but then it will not be possible by linear programming as non-linear effect of variables will be included then. Ratio of nutrients in the formulation of diet is another concern in this regard. There may be simple ratio between feed ingredients as calcium to phosphorus or complicated ratio as forage dry matter to concentrate dry matter. Also controlling the dry matter percentage as-fed in a diet involving wet feeds is a ratio constraint. Ratio constraints convert to non-linear constraints and remain outside the scope of the linear programming framework.

Variability is another matter of concern as density of certain nutrients varies considerably not only from batch to batch, but also within batches. In formulating a batch mix, it is usually specified that a minimum level of the nutrient in question will be achieved with a fixed probability. In these types of models, variance amount of each nutrient is included as a function of the inclusion rate. It assumes a non-linear relationship or a linear formulation with quadratic constraint. Special algorithms like recursive linear programming (Munford 1996) have been written for this type of problem in which problem is solved by a series of linear programming problems; effectively updating the feed matrix after each pass.

Most of the linear model models represent efficiency of converting any one of the nutrient ingredients to milk, while other represent a comparative study of effect of different nutrient ingredient on the milk yield of the animal. Thus the data available depict the linear relationship between the yield and the nutrient ingredients individually, but complexity of different nutrient ingredients would possibly be better described by nonlinear relationship between them. Non-linear programming comes to the existence to overcome these assumptions and problems. Objective of the diet problem is to find the combination of feed ingredients that will satisfy all the daily nutritional requirements of animal at minimum cost. The problem is formulated as a linear program where the objective is to minimize cost and meet constraints which require that nutritional needs be satisfied. Those constraints are included that regulate the number of calories and amounts of vitamins, minerals, fats, sodium and cholesterol in the diet. It determines the quantities of individual feedstuffs to feed to an animal in order to supply the nutrients, energy, protein and minerals required by animal. Leading to the same guideline a ration can be formulated using all its nutrient ingredients simultaneously at the optimum level. This concept of linear programming may be used to maximize the weight gain of the animal or animal yields only approximately. Introduction of nonlinear programming to optimize yield and minimize feed cost in broiler feed formulation may lead to better approximation as compared to those of linear cases. The importance of Nonlinear Programming Applications is growing due to rapidly increasing sophistication of managers and operation researchers in implementing decision oriented mathematical models, as well as to the growing availability of computer routines capable of solving large-scale nonlinear problems. While formulating a mathematical model related to real life problems, many different situations lead to nonlinear formulation of constraints and objective function. We focus on the application of nonlinear programming to the field of animal nutrition.
Nonlinear programming can be defined as:

Optimize \[ Z = f(x_1, x_2, \ldots, x_n), \]
Subject to \[ g_i(x_1, x_2, \ldots, x_n) \leq a_i, \quad i = 1, 2, 3, \ldots, m \]
And \[ x_j \geq 0, \quad j = 1, 2, 3, \ldots, n \]

Where \( f(x_1, x_2, \ldots, x_n) \) and \( g_i \) are real valued function of \( n \) decision variables and at least one of these is non-linear.

Different algorithms have been developed to solve non-linear programming problems depending upon specifications of problem.

Nonlinear models on different approaches and techniques have been derived for this purpose. Two nonlinear optimization problems arising in animal feed formulation is formulated as an iterative sequence of linear programming problems. These problems are solved by using the Ultramix feed formulation and modeling system. Ultramix is species independent system consisting of several integrated modules which include a modeler and an optimizer. Iteration is achieved by linking nutrient specifications to values obtained from the modeler, and by defining suitable stopping rules. Agway uses chance-constrained programming to formulate commercial feeds for animals. The finished feeds averaged 40 percent greater nutritional consistency and were lower in cost than feeds formulated by traditional linear programming with a margin of safety. Formulating animal feeds by chance-constrained programming saves Agway, Inc. more than $250,000 per year. Genetic algorithms are applied for the cost optimization of the feed mixtures and a software is developed by using Delphi environment, which provides flexible, extensible and user-friendly framework for tuning the heuristic relevant parameters and improving the solution quality. A stochastic-linear program Excel workbook was developed that consisted of 2 worksheets illustrating linear and stochastic program approaches. Both approaches used the Excel Solver add-in.

Proposed model is carried out to extend the work by inclusion of this nonlinear relation. This nonlinear model is constructed by considering all the three nutrient variables effect simultaneously. This technique is developed by taking three nutrient ingredients into consideration. It is seen that animal yield and weight gain are dependent on these three nutrient ingredients and can be considered as a function of these three variables. The milk yield and the efficiency with which the nutrients are utilized mainly depend on 3 factors, which may be used to maximize it. Accounting all these facts, weight gain of an animal depends upon:

1. Digestible crude protein,
2. Total digestible nutrient and;
3. Digestible dry matter.

Metabolic weight is used as a base for whole of the calculations. Studying the intake in growing animals, Moir et. Al. had earlier reported that a level between 200 and 300 K. Cal. DE per Kg0.75 is generally encountered. Individual relationship between the variables present the linear and nonlinear effects of different ingredient on body weight gain. Kuhn-Tucker conditions are used to solve this non-linear Programming problem.

This technique has been applied for three sets of data and it represents the application of nonlinear programming to maximize the body weight of animals under the given experimental conditions and satisfying NRC feeding standards. To sum up, an effort has been made in this work to give a new
dimension to the already existing multidimensional nonlinear models and its use to formulate a real world problem of optimizing the feed in terms of weight gain of the animal and to solve it as well. This objective supports the all over effect of nutrient ingredients simultaneously on the animal yield and weight gain of an animal.

It has already been accepted that nonlinear programming has a great deal of future prospects as it has direct practical utility in the field of animal nutrition.

References: